Global Optimization, Infinity Computer and Applications

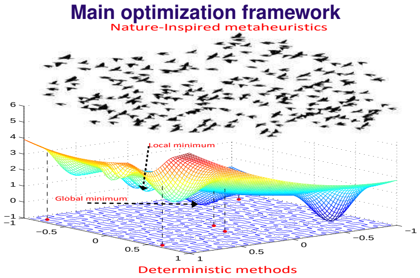
Marat S. Mukhametzhanov[1,2] and Maria Chiara Nasso[1]
[1]DIMES, University of Calabria, Italy and [2]ITMM, Nizhni Novgorod State University, Russia
{m.mukhametzhanov,mc.nasso}@dimes.unical.it

Research activity

The research consists of two fields: global optimization with expensive and noisy objective functions, and the Infinity Computer – a supercomputer allowing one to work **numerically** with finite, infinite and infinitesimal numbers in the same way. A particular attention is dedicated to solving real-life problems including the following important industrial applications: solution to expensive and ill-conditioned optimization problems in image processing and noisy data fitting; stable and precise solution to ODEs; exact higher order numerical differentiation, etc.

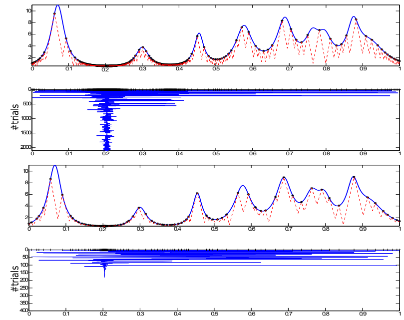
Expensive Global Optimization

Main optimization framework



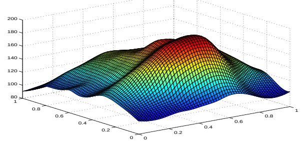
A challenging problem: given a limited computational budget, it is required to find a good approximation of the global minimum to a multiparametric and multimodal costly objective function subject to nonlinear constraints.
A promising approach: extension of univariate methods to the multivariable case by means of diagonal space-filling curves ([1, 2]).

Acceleration of Global Optimization Methods



The proposed accelerated algorithm (bottom) is more than 10 times faster with respect to the traditional algorithm (top; see [2-3]).

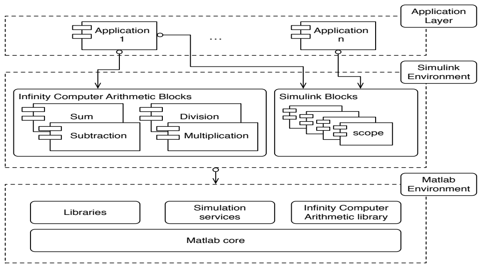
Applications in Radial Basis Function Interpolation



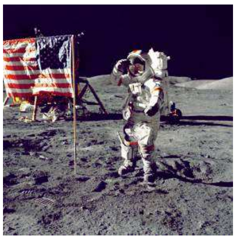
Surface of the Maunga Whau Volcano and the RBF interpolant using the shape parameter value obtained by the proposed algorithm ([4]).

The Infinity Computer

Simulink-based solution to the Infinity Computer [5]



Using ① to stars... and beyond



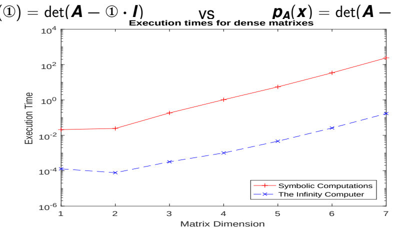
Today: using ∞

Tomorrow: using ① (see [6-7])

Infinity Computer vs Symbolic Computations

Test problem: computation of characteristic polynomials of matrices

$p_A(①) = \det(A - ① \cdot I)$ vs $p_A(x) = \det(A - x \cdot I)$

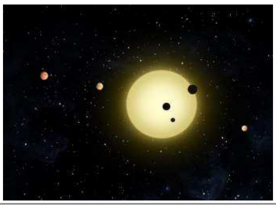


The Infinity Computer is more than 1000 times faster (see [8]).

Real-Life Applications on the Infinity Computer

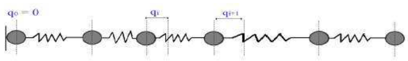
Hamiltonian Problems

Implementation of proposed and known methods on the Infinity Computer allows one to improve the accuracy and the computational effort. The methods have been tested on well-known applied problems: the Kepler problem, FPU problem, etc. (see [9] for details)



FPU problem

An illustrative application: Fermi-Pasta-Ulam problem (see [9]).



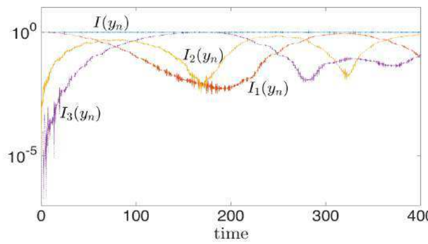
$$H(q,p) = \frac{1}{2}\sum_{i=1}^{m}(p_{2i-1}^2 + p_{2i}^2) + \frac{\omega^2}{4}\sum_{i=1}^{m}(q_{2i} - q_{2i-1})^2 + \sum_{i=0}^{m}(q_{2i+1} - q_{2i})^4$$

$$q_0 = q_{2m+1} = 0,\ p_i = \dot{q}_i,\ i = 1, \ldots, 2m, \text{ and } \omega = const.$$

The total energy $I = I_1 + \ldots + I_m$ of the linear springs is almost conserved, where

$$I_i = \frac{1}{4}\left((p_{2i} - p_{2i-1})^2 + \omega^2(q_{2i} + q_{2i-1})\right).$$

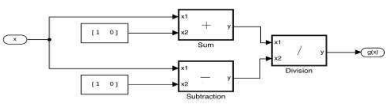
FPU problem: energy



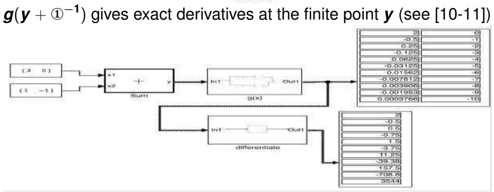
Solved by the Euler-Maclaurin methods using ① with $m = 3$.

Numerical differentiation on the Infinity Computer

$$g(x) = (x+1)/(x-1)$$

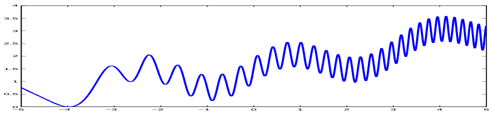


$g(y + ①^{-1})$ gives exact derivatives at the finite point y (see [10-11])

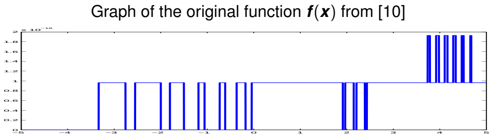


Traditional computers: ill-conditioning

Underflows/overflows in traditional systems $\rightarrow$ wrong solutions:



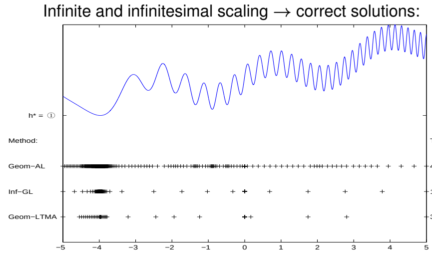
Graph of the original function $f(x)$ from [10]



Graph of the scaled function $g(x) = 10^{-16}f(x) + 1$.

Infinity Computer: well-conditioning

Infinite and infinitesimal scaling $\rightarrow$ correct solutions:



Results of the algorithms implemented on the Infinity Computer on the function $g(x) = ①^{-1}f(x) + ①$ from [12-13].

Obtained results

New powerful schemes have been proposed for optimization: visual techniques for a systematic comparison of algorithms of a different nature ([1]), acceleration techniques in derivative-free and smooth global optimization ([2-3]), a variable metric algorithm for convex non-smooth optimization ([14]). New simple and powerful higher order numerical methods for solving ordinary differential equations on the Infinity Computer have been proposed ([9-11]). The Infinity Computing has been applied to handling ill-conditioning in optimization ([12, 14]). Presented techniques can be used in different fields, where ill-conditioning appears.

References

1. Ya.D. Sergeyev, D.E. Kvasov, M.S. Mukhametzhanov, *Scientific Reports*, 8, article 453 (2018).
2. Ya.D. Sergeyev, M.S. Mukhametzhanov, D.E. Kvasov, D. Lera, *Journal of Optimization Theory and Applications*, 171(1):186 - 208 (2016).
3. D.E. Kvasov, M.S. Mukhametzhanov, M.C. Nasso, Ya.D. Sergeyev, *Lecture Notes in Computer Science*, accepted (2019).
4. M.S. Mukhametzhanov, R. Cavoretto, A. De Rossi, *Communications in Computer and Information Science*, accepted (2019).
5. A. Falcone, A. Garro, M.S. Mukhametzhanov, Ya.D. Sergeyev, *Lecture Notes in Computer Science*, accepted (2019).
6. Ya. D. Sergeyev, EU patent 1728149; RF Patent 2395111; USA patent 7,860,914.
7. Ya. D. Sergeyev, *EMS Surveys in Mathematical Sciences*, 4(2):219–320 (2017).
8. M.S. Mukhametzhanov, Ya.D. Sergeyev, *AIP Conference Proceedings*, accepted (2019).
9. F. Iavernaro, F. Mazzia, M.S. Mukhametzhanov, Ya.D. Sergeyev, *Applied Numerical Mathematics*, in Press (2019).
10. P. Amodio, F. Iavernaro, F. Mazzia, M.S. Mukhametzhanov, Ya.D. Sergeyev, *Mathematics and Computers in Simulation*, 141:24 – 39 (2017).
11. Ya.D. Sergeyev, M.S. Mukhametzhanov, F. Mazzia, F. Iavernaro, P. Amodio, *International Journal of Unconventional Computing*, 12(1):3-23 (2016).
12. Ya.D. Sergeyev, D.E. Kvasov, M.S. Mukhametzhanov, *Communications in Nonlinear Science and Numerical Simulation*, 59:319–330 (2018).
13. D.E. Kvasov, M.S. Mukhametzhanov, *Applied Mathematics and Computation*, 318:245–259 (2018).
14. M. Gaudioso, G. Giallombardo, M.S. Mukhametzhanov, *Applied Mathematics and Computation*, 318:312–320 (2018).

UNIVERSITÀ DELLA CALABRIA
DIPARTIMENTO DI INGEGNERIA INFORMATICA, MODELLISTICA, ELETTRONICA E SISTEMISTICA (DIMES)
Dipartimento di Eccellenza 2018-2022

LOBACHEVSKY UNIVERSITY

iNdAM Istituto Nazionale di Alta Matematica Francesco Severi